Scalability of Bidirectional Vehicle Strings with Static and Dynamic Measurement Errors

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Abstract

Poor scalability arises in many vehicle platoon problems. Bidirectional strings appear to show some promise for mitigating these problems. In some cases these solutions have the undesirable side effect of non-scalable response to measurement errors. We examine this problem and show how information exchange between neighbouring vehicles may eliminate scalability difficulties due to measurement errors.

Key words: scalability, string stability, vehicle strings, measurement errors, measurement noise

1 Introduction

In the simplest form of formation control, a group of N vehicles (e.g. "platoon" or "string") is required to move in one direction. While the first vehicle follows a reference trajectory, the remaining vehicles aim to keep a prescribed distance to neighbouring vehicles. It is desirable to find distributed control solutions, using local measurements only. In this paper bidirectional distributed control of a string is studied. The string is referred to as "bidirectional" when the local controller uses information of preceding and following vehicles, Barooah et al. (2009), in contrast to "unidirectional" where only the state of a group of preceding vehicles is considered, Klinge & Middleton (2009). Using a bidirectional structure is motivated by ease of implementation as data from two surrounding vehicles can be easily measured by onboard sensors. Another main advantage is that a weaker form of "string stability" can be guaranteed without using a time headway, Knorn et al. (2014*a*). (For time headway see Chien & Ioannou (1992).)

"String instability" ("slinky effect", Zhang et al. (1999), or 'unscalability', Lestas & Vinnicombe (2006)) describes the case when small error signals are amplified when travelling through the string resulting in growth of the local error norm with the position in the string. The widely used definition of string stability requires the l_2 norm of the state vector to be uniformly bounded in the presence of l_2 disturbances, Seiler et al. (2004), Barooah & Hespanha (2005). It was shown in Seiler et al. (2004), Barooah & Hespanha (2005) that linear symmetric bidirectional strings with two integrators in the open loop and constant spacing are always string unstable if no global information is available. Lestas & Vinnicombe (2007) showed that string stability can be achieved with sufficiently large coupling with the leader position. Knorn et al. (2013, 2014*a*) extended the idea in Eyre et al. (1998) of modelling a symmetric bidirectional string as a mass-spring-damper system, and derived sufficient conditions to guarantee a weaker form of string stability.

However, most results assume perfect measurements of all states. Communication delays between the lead vehicle and the rest of the string were considered in Huang & Ren (1997), Liu et al. (2001), Xiao et al. (2008, 2009), Öncü et al. (2011), Guo & Yue (2012), Peters et al. (2014). Guo & Yue (2014) proposed string stable controllers for unidirectional

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strings subject to delays and sensor failures. The effects of communication losses have been studied in Seiler & Sengupta (2001), Glavaški et al. (2003), Cao (2010). But string stability was not discussed.

We study a system with offsets and noise in the inter vehicle distance measurements. When assuming different measuring errors for the same distance, the errors might lead to string instability. This problem can be avoided if a simple consensus algorithm is implemented. Preliminary results were presented in Knorn et al. (2014*b*). After the system description in Section 2, the effects of measurement offsets and noise will be discussed in Section 3 and 4, respectively. Numerical examples can be found in Section 5.

Notation: The state, steady state and disturbance vectors are $x(t) = (x_1(t),\dots,x_N(t))^{\mathrm{T}}$, $x_0 = (x_{1_0},\dots,x_{N_0})^{\mathrm{T}}$ and $d(t) = (d_1(t),\dots,d_N(t))^{\mathrm{T}}$. The vector of ones is $\underline{1}$ and $\underline{e}_i$ is the ith canonical vector. The diagonal matrix $A \in \mathbb{R}^{N\times N}$ with entries $a_1,\dots a_N$ is $A = \mathrm{diag}(a_1,\dots a_N)$. The L_2 vector norm is $|x|_2 = |x| = \sqrt{x^{\mathrm{T}}x}$ and the l_2 and l_∞ norms are $\|x(\cdot)\|_2 = \sqrt{\int_0^\infty |x(t)|^2 \mathrm{d}t}$ and $\|x(\cdot)\|_\infty = \sup_{t\geq 0}|x(t)|$, respectively.

2 System Description

The mass, momentum and position of the *ith* vehicle are m_i, p_i and q_i respectively. Hence, for $i = 1,2,\cdots,N$: $\dot{p}_i = F_i + d_i$ and $\dot{q}_i = m_i^{-1}p_i$ where d_i is the disturbance and F_i is the control force. The local control objective is to bring the local error to zero via distributed control depending only on local measurements and short range communication. The local error is defined as a linear combination of position errors towards a limited group of direct predecessors and followers. The controller for the first vehicle in the string aims to follow a given trajectory $q_0 = v_0 t$ and minimise the local position error towards a group of following vehicles. Neither the reference position, velocity v_0, nor any data of the leading vehicle are known by other vehicles in the string apart from a small number of vehicles at the beginning of the string. The global control objective is to achieve "l_2 weak string stability":

Definition 1 *The equilibrium x^* of a system with N agents is l_2 weakly string stable with respect to disturbances $d(t)$, if given any $\epsilon > 0$, there exist $\delta_1(\epsilon) > 0$ and $\delta_2(\epsilon) > 0$ such that $|x(0) - x^*| < \delta_1(\epsilon)$ and $\|d(\cdot)\|_2 < \delta_2(\epsilon)$ implies $\|x(\cdot) - x^*\|_\infty = \sup_{t\geq 0}|x(t) - x^*| < \epsilon$ for all $N \geq 1$.*

In the literature, different definitions for "string stability" are known. For instance the definition in Seiler et al. (2004), Barooah & Hespanha (2005) requires the l_2 norm of $x(t)-x^*$ to be bounded independently of N. Since it was shown in Barooah & Hespanha (2005) that strings of the form studied here are always string unstable (according to the standard definition), a weaker form of string stability using the l_∞ norm of the deviations is defined here.

Choosing the local errors $\Delta_i = q_{i-1} - q_i$ as system states yields $\dot{\Delta}_i = \dot{q}_{i-1} - \dot{q}_i = m_{i-1}^{-1}p_{i-1} - m_i^{-1}p_i$. Thus,

$$\begin{bmatrix}\dot{p}\\ \dot{\Delta}\end{bmatrix} = \begin{bmatrix}0 & S^{\mathrm{T}}\\ -S & 0\end{bmatrix}\nabla H(p,\Delta) + \begin{bmatrix}F\\ 0\end{bmatrix} + \begin{bmatrix}d\\ \underline{e}_1 v_0\end{bmatrix}, \tag{1}$$

where $\Delta = (\Delta_1,\dots,\Delta_N)^{\mathrm{T}}$, $p = (p_1,\dots,p_N)^{\mathrm{T}}$, and $F = (F_1,\dots,F_N)^{\mathrm{T}}$, $H(p,\Delta) = \frac{1}{2}p^{\mathrm{T}}M^{-1}p$, $M = \mathrm{diag}(m_1,\dots,m_N)$ and S has the bidiagonal form with "1" as diagonal entries, "-1" as entries directly below the diagonal and zero entries otherwise. First, consider $F = -(B+R)M^{-1}p + \underline{e}_1 R_1 v_0 + S^{\mathrm{T}}C^{-1}\Delta$, with virtual spring forces between the vehicles (including the reference "0"), which are parametrised by $C = \mathrm{diag}(c_1,\dots,c_N)$ where $c_i > 0$ is the compliance of the spring between vehicles $i-1$ and i; virtual dampers between the vehicles (including the reference "0") described by

$$R = \begin{bmatrix} R_1+R_2 & -R_2 & 0 & \cdots & 0\\ -R_2 & R_2+R_3 & -R_3 & \ddots & \vdots\\ 0 & -R_3 & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & R_{N-1}+R_N & -R_N\\ 0 & \cdots & 0 & -R_N & R_N \end{bmatrix} \tag{2}$$

where $R_i > 0$ is the damping coefficient between vehicles $i-1$ and i, and the drag forces described by $B = \mathrm{diag}(b_1,\dots,b_N)$ with $b_i > 0\ \forall i$. Further, integral action control F_{IA} is added:

$$F = -(B+R)M^{-1}p + \underline{e}_1 R_1 v_0 + S^{\mathrm{T}}C^{-1}\Delta + F_{\mathrm{IA}} \tag{3}$$

$$F_{\mathrm{IA}} = MKS^{\mathrm{T}}C^{-1}\Delta - (B+R)Kz_3 \tag{4}$$

$$\dot{z}_3 = -S^{\mathrm{T}}C^{-1}\Delta. \tag{5}$$

F_{IA} was designed using port-Hamiltonian theory, Ortega & Garcia-Canseco (2004), Donaire & Junco (2009), Ortega & Romero (2012). z_3 is the vector of controller states and the integral action control parameter is $K = \mathrm{diag}(k_1,\dots,k_N)$.

Theorem 2 *Knorn et al. (2014*a*): Consider* (1), (3)-(5)*. Assume d includes a constant component d_{c} and a dynamical component $d_{\mathrm{d}}(t)$ such that $d = d_{\mathrm{c}} + d_{\mathrm{d}}(t)$ and there exists a constant D s.t. $\|d_{\mathrm{d}}(\cdot)\|_2 \leq D < \infty$ for all N. Then,*

(1) $(p^*,\Delta^*,z_3^*) = \left(M\underline{1}v_0, 0, K^{-1}(B+R)^{-1}(d_{\mathrm{c}} - B\underline{1}v_0)\right)$ is globally asymptotically stable for $d_{\mathrm{d}} = 0$ (despite d_{c}),

(2) the system is passive with input $d_{\mathrm{d}}(t)$, output $y = \nabla_{z_1}H_z(z)$ and storage function $H_z(z)$, and

(3) the system is l_2 weakly string stable w.r.t. $d_{\mathrm{d}}(t)$.

This implies that the control signals are bounded independently of N if all parameters are chosen within some fixed bounds since the control is a linear function of the states.

3 Constant Measurement Offsets

Δ_i is measured by vehicles i and $i-1$. Vehicle $i-1$ measures its back distance $\Delta_{\mathrm{m,b},i} = \Delta_i + \hat{\Delta}_{\mathrm{b},i}$ with the offset $\hat{\Delta}_{\mathrm{b},i}$ and vehicle i measures its front distance $\Delta_{\mathrm{m,f},i} = \Delta_i + \hat{\Delta}_{\mathrm{f},i}$ with the offset $\hat{\Delta}_{\mathrm{f},i}$. The measurement vectors are

$$\Delta_{\mathrm{m,f}} = \Delta + \hat{\Delta}_{\mathrm{f}} \quad \text{and} \quad \Delta_{\mathrm{m,b}} = \Delta + \hat{\Delta}_{\mathrm{b}}. \tag{6}$$

with $\hat{\Delta}_{\mathrm{f}} = \left(\hat{\Delta}_{\mathrm{f},1},\dots,\hat{\Delta}_{\mathrm{f},N}\right)^{\mathrm{T}}$, $\hat{\Delta}_{\mathrm{b}} = \left(0,\hat{\Delta}_{\mathrm{b},2},\dots,\hat{\Delta}_{\mathrm{b},N}\right)^{\mathrm{T}}$. Then,

$$\hat{\Delta} := C^{-1}\hat{\Delta}_{\mathrm{f}} + (S^{\mathrm{T}} - I)C^{-1}\hat{\Delta}_{\mathrm{b}}. \tag{7}$$

If communication between neighbouring vehicles is possible, both agents can agree on one measurement. The algebraic mean of the two measurements can be used (measurement consensus) or the distance can be measured at one vehicle and communicated to the other vehicle. In this case, $\hat{\Delta}_\mathrm{f} = \hat{\Delta}_\mathrm{b}$ and (7) yields $\hat{\Delta} = S^\mathrm{T}C^{-1}\hat{\Delta}_\mathrm{f} = S^\mathrm{T}C^{-1}\hat{\Delta}_\mathrm{b}$.

If the measurements are subject to offsets, the system settles on a different equilibrium as in Theorem 2. Even though the equilibrium is asymptotically stable, the distances in steady state might grow without bound as $N \to \infty$.

Lemma 3 *Consider* (1), (3)-(5). *If the system is void of dynamic disturbances $d_\mathrm{d}(t)$ and the measurements are subject to offsets as in* (6), *then* $\left(p^*,\Delta^*,z_3^*\right) = \left(M\underline{1}v_0, -CS^{-\mathrm{T}}\hat{\Delta},\alpha\right)$

$$\textit{with} \quad \alpha = K^{-1}(B+R)^{-1}\left(d_\mathrm{c} - B\underline{1}v_0\right) \tag{8}$$

and $\hat{\Delta}$ in (7) *is the stable equilibrium of the system.*

PROOF. If each vehicle uses its measurement, the spring forces are $C^{-1}\Delta_\mathrm{m,f}+\left(S^\mathrm{T}-I\right)C^{-1}\Delta_\mathrm{m,b} = S^\mathrm{T}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right)$, and (3)-(5) yield $F = \underline{e}_1R_1v_0 + S^\mathrm{T}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right) - (B+R)M^{-1}p + F_\mathrm{IA}$, $F_\mathrm{IA} = MKS^\mathrm{T}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right) - (B+R)Kz_3$, $\dot{z}_3 = -S^\mathrm{T}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right)$. Setting $z_1 = p - M\underline{1}v_0 + MK(z_3-\alpha)$ and $z_2 = \Delta + CS^{-\mathrm{T}}\hat{\Delta}$ yields

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}-(B+R) & S^\mathrm{T} & 0\\ -S & 0 & S\\ 0 & -S^\mathrm{T} & 0\end{bmatrix}\nabla H_\mathrm{z}(z) \quad \text{with} \tag{9}$$

$$H_\mathrm{z}(z) = \frac{1}{2}z_1^\mathrm{T}M^{-1}z_1 + \frac{1}{2}z_2^\mathrm{T}C^{-1}z_2 + \frac{1}{2}(z_3-\alpha)^\mathrm{T}K(z_3-\alpha). \tag{10}$$

Using $H_\mathrm{z}(z)$ as Lyapunov function, yields $\dot{H}_\mathrm{z}(z) = -\nabla_{z_1}^\mathrm{T}H_\mathrm{z}(z)(B+R)\nabla_{z_1}H_\mathrm{z}(z) \leq 0$ since $(B+R) > 0$. The biggest invariant set included in $\mathcal{S} = \{z|\dot{H}_\mathrm{z}(z) = 0\}$ is $(z_1^*,z_2^*,z_3^*) = (0,0,\alpha)$. Thus, by LaSalle's Invariance Principle, (Khalil 2001, Thm 4.4), the equilibrium (z_1^*,z_2^*,z_3^*) is asymptotically stable. This implies the result. □

Remark 4 *The equilibrium of the displacements is* $\Delta^* = -CS^{-\mathrm{T}}(C^{-1}\hat{\Delta}_\mathrm{f} + (S^\mathrm{T} - I)C^{-1}\hat{\Delta}_\mathrm{b})$. *This implies* $\Delta_i^* = \sum_{k=i+1}^N \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{b},k} - \sum_{k=i}^N \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{f},k}$. *If $\hat{\Delta}_{\mathrm{f},i} > \hat{\Delta}_{\mathrm{b},i}$ for all i, then the distance between the first two vehicles gets smaller when the string size increases. Thus, for an increasing string length the cars at the beginning of the string crash. This effect could be reduced by choosing decreasing c_i towards the end of the string. However, c_i have to decrease drastically and in fact approach zero as N increases. This is clearly undesirable in practise.*

Remark 5 *One might assume that* on average $\hat{\Delta}_{\mathrm{b},i} = \hat{\Delta}_{\mathrm{f},i}$. *However, even if the expected value of the steady state error does not grow, its variance still grows without bound as N increases: Assume all offsets are uncorrelated, have an expected value of* 0 *and a variance of* $\mathrm{Var}\left(\hat{\Delta}_{\mathrm{b},i}\right) = \mathrm{Var}\left(\hat{\Delta}_{\mathrm{f},i}\right) = \sigma^2 < \infty$ *for all i. Then,* $\mathrm{Var}\left(\Delta_i^*\right) = \mathrm{Var}\left(\sum_{k=i+1}^N \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{b},k}\right) + \mathrm{Var}\left(\sum_{k=i}^N \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{f},k}\right) = \sigma^2\left(2\sum_{k=i+1}^N\left(\frac{c_i}{c_k}\right)^2 + 1\right)$. *For $i = 1$ and $c_k = c$ for all k,* $\mathrm{Var}\left(\Delta_1^*\right)$ *grows linearly with N.*

When simple measurement consensus can be reached using basic inter-vehicle communication, stability of a bounded equilibrium can be guaranteed:

Theorem 6 *Consider* (1), (3)-(5). *Assume the measurements are subject to offsets and simple measurement consensus is reached such that* $\hat{\Delta} := S^\mathrm{T}C^{-1}\hat{\Delta}_\mathrm{f} = S^\mathrm{T}C^{-1}\hat{\Delta}_\mathrm{b}$. *If there exists a $\delta < \infty$ such that $|\hat{\Delta}_{\mathrm{f},i}| = |\hat{\Delta}_{\mathrm{b},i}| < \delta$ for all i, then the equilibrium* $(p^*,\Delta^*,z_3^*) = (M\underline{1}v_0, -CS^{-\mathrm{T}}\hat{\Delta},\alpha)$ *is bounded and asymptotically stable.*

PROOF. Due to Lem. 3, $(p^*,\Delta^*,z_3^*) = (M\underline{1}v_0, -CS^{-\mathrm{T}}\hat{\Delta},\alpha)$. Thus, $\Delta^* = \hat{\Delta}_\mathrm{f} = \hat{\Delta}_\mathrm{b}$. Due to δ, there exists an upper bound on Δ^*. According to (Knorn et al. 2013, Lem. 2) also all entries of $z_3^* = \alpha$ are bounded independently of N. □

Hence, a bounded equilibrium can be guaranteed by establishing a simple consensus algorithm between neighbouring vehicles despite unknown measurement offsets. If the offsets for both vehicles are equivalent, they do not accumulate at the beginning of the string, and the displacements in steady state are bounded if all offsets are bounded.

4 Time Varying Measurement Noise

Assume the distance measurements are subject to measurement noise with zero mean described by $\check{\Delta}_{\mathrm{f},i}(t)$ or $\check{\Delta}_{\mathrm{b},i}(t)$ for forward or backwards measurements, respectively. Thus,

$$\Delta_\mathrm{m,f} = \Delta + \check{\Delta}_\mathrm{f}(t) \quad \text{and} \quad \Delta_\mathrm{m,b} = \Delta + \check{\Delta}_\mathrm{b}(t) \tag{11}$$

where $\check{\Delta}_\mathrm{f}(t) = \left(\check{\Delta}_{\mathrm{f},1}(t), \check{\Delta}_{\mathrm{f},2}(t), \ldots, \check{\Delta}_{\mathrm{f},N}(t)\right)^\mathrm{T}$ and $\check{\Delta}_\mathrm{b}(t) = \left(0, \check{\Delta}_{\mathrm{b},2}(t), \ldots, \check{\Delta}_{\mathrm{b},N}(t)\right)^\mathrm{T}$. Similar to (7), define

$$\check{\Delta}(t) := C^{-1}\check{\Delta}_\mathrm{f}(t) + (S^\mathrm{T} - I)C^{-1}\check{\Delta}_\mathrm{b}(t). \tag{12}$$

If measurement consensus is not reached, string instability can be shown:

Lemma 7 *Consider* (1), (3)-(5). *Assume the distance measurements are subject to noise as in* (11), *where there exists a $\mu < \infty$ such that $|\check{\Delta}_{\mathrm{f},i}(t)| \leq \mu$ and $|\check{\Delta}_{\mathrm{b},i}(t)| \leq \mu$ for all $i > 0$ and $t \geq 0$. Then the system is not l_2 weakly string stable.*

PROOF. Subject to noise, equations (3)-(5) change to $F = -(B+R)M^{-1}p + \underline{e}_1R_1v_0 + S^\mathrm{T}C^{-1}(\Delta + CS^{-\mathrm{T}}\check{\Delta}(t)) + F_\mathrm{IA}$, $F_\mathrm{IA} = MKS^\mathrm{T}C^{-1}(\Delta + CS^{-\mathrm{T}}\check{\Delta}(t)) - (B+R)Kz_3$, $\dot{z}_3 = -S^\mathrm{T}C^{-1}(\Delta + CS^{-\mathrm{T}}\check{\Delta}(t))$ with $\check{\Delta}(t)$ in (12). Choosing $z_1 = p - M\underline{1}v_0 + MK(z_3-\alpha)$ and $z_2 = \Delta + CS^{-\mathrm{T}}\check{\Delta}(t)$ yields

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}-(B+R) & S^\mathrm{T} & 0\\ -S & 0 & S\\ 0 & -S^\mathrm{T} & 0\end{bmatrix}\nabla H_\mathrm{z}(z) + \begin{bmatrix}0\\ CS^{-\mathrm{T}}\dot{\check{\Delta}}(t)\\ 0\end{bmatrix} \tag{13}$$

with (10). To show that the system subject to noise is string unstable it suffices to show that there exists at least one noise vector meeting the allowed specifications such that the states of the system grow with the string size N. Assume $c_i = c$ and $\breve{\Delta}_{\mathrm{b},i}(t) = -\breve{\Delta}_{,i}(t) = \mu/\omega \sin(\omega t)$ for all i. Then, the ith entry of $CS^{-\mathrm{T}}\breve{\Delta}(t)$ is $(\sum_{k=i+1}^{N}\cos(\omega t) + \sum_{k=i}^{N}\cos(\omega t))\mu = (2(N-i)+1)\cos(\omega t)\mu$. If the response of vehicle 1 to $\cos(\omega t)\mu$ is denoted by $y_1(t)$, then its response to the noise vector chosen above is $(2N-1)y_1(t)$. Thus, the deviations at the beginning of the string grow without bound as N increases.□

l_2 weak string stability can be guaranteed if measurement consensus is reached between neighbouring vehicles:

Theorem 8 *Consider* (1), (3)-(5). *Assume the distance measurements are subject to noise and measurement consensus is reached such that* $\breve{\Delta}(t) := S^{\mathrm{T}}C^{-1}\breve{\Delta}_{\mathrm{f}}(t) = S^{\mathrm{T}}C^{-1}\breve{\Delta}_{\mathrm{b}}(t)$. *If the vector of measurement noise* $\breve{\Delta}(t)$ *is in* l_2 *uniformly in* N, *that is, there exist a* $\kappa < \infty$ *such that for all* N: $\|\breve{\Delta}(\cdot)\|_2 \le \kappa$, *then the equilibrium* $(p^*, \Delta^*, z_3^*) = (M\underline{1}v_0, 0, \alpha)$ *is asymptotically stable and the system is* l_2 *weakly string stable.*

PROOF. Set $z_1 = p - M\underline{1}v_0 + MK(z_3 - \alpha)$ and $z_2 = \Delta$. Then, the closed loop dynamics have the port-Hamiltonian form

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}-(B+R) & S^{\mathrm{T}} & 0\\ -S & 0 & S\\ 0 & -S^{\mathrm{T}} & 0\end{bmatrix}\nabla H_z(z) + \begin{pmatrix}d_{\mathrm{d}}(t) + \breve{\Delta}(t)\\ 0\\ -\breve{\Delta}(t)\end{pmatrix} \tag{14}$$

with the Hamiltonian function (10). $z(t)$ can be written as $z(t) = z_{\mathrm{a}}(t) + z_{\mathrm{b}}(t)$ where $z_{\mathrm{a}}(t)$ and $z_{\mathrm{a}}(t)$ are the responses of the system to $(d_{\mathrm{d}}^{\mathrm{T}}(t) + \breve{\Delta}(t)^{\mathrm{T}}, 0, 0)^{\mathrm{T}}$ and $-(0, 0, \breve{\Delta}(t)^{\mathrm{T}})^{\mathrm{T}}$, respectively. Since $\breve{\Delta}(t)$ is in l_2, Thm. 2 shows that $\|z_{\mathrm{a}}(\cdot) - z_{\mathrm{a}}^*\|_\infty$ is bounded independently of N for all $d_{\mathrm{d}}(t)$ in l_2. To bound the norm of $\|z_{\mathrm{b}}(\cdot) - z_{\mathrm{b}}^*\|_\infty$, note that the autonomous system is globally asymptotically stable, Thm. 2. Hence, the matrix in the first right hand term of (14) (below denoted by A) is Hurwitz. Thus, for all N there exist $k < \infty$, $\lambda < 0$ such that

$$\left|z_{\mathrm{b}}(t) - z_{\mathrm{b}}^*\right|^2 \le k\left|z_{\mathrm{b}}(0) - z_{\mathrm{b}}^*\right|^2 + k\int_0^t \left|\breve{\Delta}(t)\right|^2 d\tau. \tag{15}$$

Using Geršgorin's Theorem, e. g. Horn & Johnson (1985), it can be shown that all eigenvalues of A are bounded independently of N. Hence, there exists a $k < \infty$ for all N. Further, as N grows, the minimal eigenvalue of A approaches 0, which yields $\lambda \to 0$ as $N \to \infty$. (15) holds in the limit. Also note that the last term in (15) is bounded by the l_2 norm of $\breve{\Delta}(t)$ squared which is assumed to be bounded for all N. Hence, $|z_{\mathrm{b}}(t) - z_{\mathrm{b}}^*|^2$ is bounded independently of N for all t. □

5 Example

In the first scenario two homogeneous bidirectional vehicle strings with $N = 10$ and $N = 100$ have been simulated. In both cases the reference trajectory is $q_0 = v_0 t$ and all vehicles start with zero initial values. The measurements are

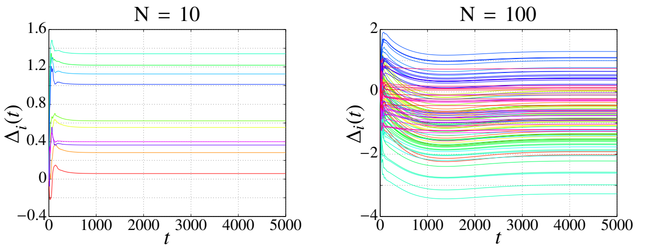

Figure 1. Scenario 1: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, N (purple)

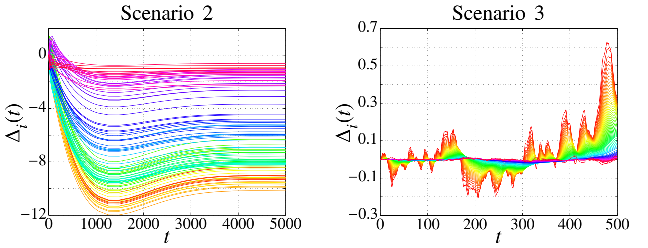

Figure 2. Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, $N = 100$ (purple)

subject to random distinct forward and backwards measurement offsets between $[-0.5, 0.5]$. While it seems in Fig. 1 that measurement offsets are accumulating in steady state for $N = 10$, close examination of the case $N = 100$ reveals that this is not the case as the string length increases. However, the span of the steady state displacements grows from approximately $0 - 1.4$m for $N = 10$ to $-3.5 - 1.5$m for $N = 100$.

In Scenario 2, an additional offset of 0.1m is added to each forward measurement error. As expected, the steady state deviations of Δ at the beginning of the string increase, Fig. 2. The effect of noise without measurement consensus is illustrated in scenario 3. Here, random measurement noise $n(t)$ in between $-0.05m$ and $0.05m$ is generated. To illustrate the worst case, the forward and backward measurement errors are chosen $\breve{\Delta}_{\mathrm{f},i} = -\breve{\Delta}_{\mathrm{b},i} = n(t)$ for i. As shown in Fig. 2 this leads to an increasing variation of the displacements between the vehicles at the beginning of the string.

In the last scenario, it is assumed that measurement consensus is reached. In this case, the measurement offsets no longer accumulate at the beginning of the string, and the variance of the steady state deviations does not increase. Note that the effects of the measurement noise in Fig. 3 are much smaller than in Senario 3 but still partly accumulate at the beginning of the string. This is because $n(t)$ is not in l_2.

6 Conclusions

This paper studies the effect of measurement offsets and noise on string stability of bidirectional vehicle platoons. It is shown that in case the constant measurement offsets differ between neighbouring vehicles measuring the same distance, the errors might accumulate at the beginning of the string. When assuming time-varying measurement noise, it can be shown that asymmetry in the measurement noise between neighbouring vehicles leads to string instability. In both cases, the negative effects can be avoided by implementing a simple consensus algorithm between the vehicles

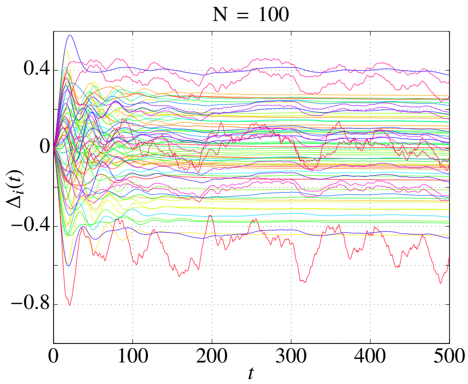

Figure 3. Scenario 4: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, 100 (purple)

such that l_2 weak string stability is guaranteed. A drawback of the simple consensus algorithms is the fact that wireless communication is prone to communication losses and delays, whose effects should be investigated in the future.

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